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EVALUATING APPROXIMATE MEAN AND VARIANCE OF ESTIMATED RELIABILITY IN INTERFERENCE MODELS USING MONTE-CARLO SIMULATION (MCS)

A. N. PATOWARY[1], G. L. SRIWASTAV[2] & J. HAZARIKA[3]
[1]College of Fisheries, Assam Agricultural University, Assam, India
[2,3]Department of Statistics, Dibrugarh University, Assam, India

ABSTRACT

To obtain the expressions for mean and variance of reliability estimate, $\hat{R} = P(X \geq Y)$, analytically, is generally difficult. Here, we find approximate expressions for mean and variance of estimated system reliability in interference theory when stress and strength follow some particular distributions. We have evaluated approximates of mean and variance of estimated reliability when stress-strength both follows either exponential or normal distribution. For validity of approximation method, we have used Monte-Carlo simulation. Also, Normal probability plots of estimated reliability samples are drawn for different values of the parameters of the distribution. From Monte Carlo simulation (MCS), it is observed that approximation for mean and variance of estimated reliability is up to the mark.

KEYWORDS: Reliability, Interference Theory, Monte-Carlo Simulation (MCS)

1. INTRODUCTION

In Interference theory of reliability R=P(X$\geq$Y), the reliability of a system and its other reliability characteristics can be expressed as some functions of the parameters of the distributions of strength (X) and stress (Y) associated with the functioning of the system. We estimate these parameters and substitute these values in the expressions for reliability and other characteristics to get their estimates. If the estimates of parameters used here are maximum likelihood estimators then from the invariance property of MLE's, the corresponding estimators of reliability are also MLE's. There exists extensive literature for estimation of reliability analytically for single component systems. But the reliability expressions for multi-component systems are not simple enough to facilitate analytical estimation of reliability and its other characteristics. Also, due to lack of stress-strength data one way out is simulation. For example, (Manders et al., 1982, Aldrisi, 1987, Stumpf and Schwartz 1993, Zhang et al., 2010) have simulated stress-strength and estimated reliability. (Paul and Borhanuddin, 1997, Rezaei et al., 2010) estimated reliability of stress-strength model, using Monte-Carlo simulation (MCS). (Ahmad et al., 1997), obtain Bayes estimates of P(Y<X) using MCS. (Uddin et al., 1993) estimated reliability for multicomponent system using MCS. (Patowary et al., 2012) estimated reliability of n-standby system using Monte-Carlo simulation. Similarly, it is difficult to obtain the distribution of a reliability estimator or even its exact mean and variance, analytically. In this paper, an attempt has been made to find approximate expressions for mean and variance of estimated reliability on the basis of (Lyold and Lipow, 1962) and check the validity of the approximation by Monte-Carlo simulation.

In Section 2, we have given the approximate expressions for mean and variance of single parameter and two parameters distribution cases. In Section 3, approximation expressions of mean and variance are obtained when stress-strength (S-S) both follows exponential or normal distribution. Also, Monte-Carlo simulation (MCS) is extensively performed for verification purpose of expressions.

2. METHOD TO OBTAIN MEAN AND VARIANCE OF ESTIMATED RELIABILITY

In this paper, existing method of (Lloyd and Lipow, 1962) is used to estimate reliability in interference models when stress-strength follows particular distribution. However, as it is not easy to get real life data, we have used MCS for estimation of mean and variance of estimated reliability.

The reliability can be expressed as a function of the parameters $\tilde{\theta} = \theta_1, \ \theta_2, \ \ldots$ of the distributions of strength X and stress Y. According to the above method, if $\hat{R}\,(\tilde{\theta})$ is the MLE of R ($\tilde{\theta}$), then

(i) If $\tilde{\theta} = \theta$ i.e only one parameter is involved, then the approximate mean of $\hat{R}(\hat{\theta})$ is given by

$$E\left[\hat{R}(\hat{\theta})\right] = R\,(\theta) + O\left(\frac{1}{M}\right), \text{ when } E\,(\hat{\theta}) \simeq \theta, \tag{2.1}$$

Where $O\left(\frac{1}{M}\right) \to 0$ as $M \to \infty$.

M is the sample size of estimated reliability samples. Obviously, $\hat{R}(\hat{\theta})$ is asymptotically unbiased for R(θ).

The approximate variance of $\hat{R}(\hat{\theta})$ is given by

$$\text{Var}\left[\hat{R}(\hat{\theta})\right] = \left[\frac{\partial \hat{R}(\hat{\theta})}{\partial \hat{\theta}}\right]_{\hat{\theta}=\theta} \text{Var}\,(\hat{\theta}) + O\left(\frac{1}{M^{3/2}}\right) \tag{2.2}$$

where $O\left(\frac{1}{M^{3/2}}\right) \to 0$ as $M \to \infty$.

(ii) If $\tilde{\theta} = (\lambda, \mu)$, i.e., two parameters case, then the approximate mean and variance of $\hat{R}\,(\hat{\lambda}, \hat{\mu})$ are given, respectively, by Eq.(2.3) and Eq.(2.4)

$$E\left[\hat{R}(\hat{\lambda}, \hat{\mu})\right] = R(\lambda, \mu) + O\left(\frac{1}{M}\right), \tag{2.3}$$

when $E(\hat{\lambda}) \simeq \lambda$ and $E(\hat{\mu}) \simeq \mu$.

So, $\hat{R}\,(\hat{\lambda}, \hat{\mu})$ is asymptotically unbiased for $R(\lambda, \mu)$.

$$\text{And Var}\left[\hat{R}(\hat{\lambda}, \hat{\mu})\right] = \left[\frac{\partial \hat{R}(\hat{\lambda}, \hat{\mu})}{\partial \hat{\lambda}}\right]^2_{\hat{\lambda}=\lambda, \hat{\mu}=\mu} \text{Var}(\hat{\lambda}) + \left[\frac{\partial \hat{R}(\hat{\lambda}, \hat{\mu})}{\partial \hat{\mu}}\right]^2_{\hat{\lambda}=\lambda, \hat{\mu}=\mu} \text{Var}(\hat{\mu})$$

$$+\left[\frac{\partial \hat{R}(\hat{\lambda},\hat{\mu})}{\partial \hat{\lambda}}\right]_{\hat{\lambda}=\lambda,\hat{\mu}=\mu}\left[\frac{\partial \hat{R}(\hat{\lambda},\hat{\mu})}{\partial \hat{\mu}}\right]_{\hat{\lambda}=\lambda,\hat{\mu}=\mu} \operatorname{Cov}(\hat{\lambda},\hat{\mu}) + O\left(\frac{1}{M^{3/2}}\right), \tag{2.4}$$

Let DM be the difference between $E\left[\hat{R}(\hat{\lambda},\hat{\mu})\right]$ and $R(\lambda,\mu)$ i.e.

$$DM = \left|E\left[\hat{R}(\hat{\lambda},\hat{\mu})\right] - R(\lambda,\mu)\right| \tag{2.5}$$

Similarly, let DV be the difference of $Var\left[\hat{R}(\hat{\lambda},\hat{\mu})\right]$ and r.h.s of Eq.(2.4).

If the values of these differences are negligible, then approximate expressions are considered as satisfactory.

3. STRESS-STRENGTH FOLLOWS PARTICULAR DISTRIBUTION

We have seen that in interference models system reliability is a function of stress-strength parameters. Let f(x) be the p.d.f. of strength (X) of the system and g(y) be that of the stress (Y) on the system. Here, we have considered two cases.

Case I: When both f(x) and g(y) follows exponential distribution (one parameter case)

Case II: When both f(x) and g(y) follows normal distribution (two parameter case)

3.1 Stress-Strength Exponentially Distributed

Let f(x) and g(y) be exponential with means λ and μ, respectively. Then the system reliability is given by (Kapur and Lamberson, 1977)

$$R = \frac{\lambda}{\lambda+\mu} = R(\lambda,\mu), \text{ say, } \lambda > 0,\ \mu > 0 \tag{3.1.1}$$

Suppose M units are put on test. Let $x_1, x_2, \ldots, x_M$ be the strengths of the M units and let $y_1, y_2, \ldots, y_M$ be the stresses working on them. Now we know that for exponential distribution sample mean is an MLE of population mean, and is unbiased, consistent, sufficient and efficient. Hence, $\bar{x}\left(=\frac{1}{M}\sum x_i\right)$ and $\bar{y}\left(=\frac{1}{M}\sum y_i\right)$ are MLE's of λ and μ with the same properties,

$$\text{i.e. } \hat{\lambda} = \bar{x} \text{ and } \hat{\mu} = \bar{y} \tag{3.1.2}$$

An estimate of R is given by

$$\hat{R} = \frac{\bar{x}}{\bar{x}+\bar{y}} = \hat{R}(\bar{x},\bar{y}), \text{ say} \tag{3.1.3}$$

Since, $\hat{R}$ is a one-valued function of $\bar{x}$ and $\bar{y}$, hence from the properties of MLE's, $\hat{R}$ is an MLE of R. So from Eq.(2.3)

$$E\left[\hat{R}(\bar{x},\ \bar{y})\right] = R(\lambda,\ \mu) + O\left(\frac{1}{M}\right), \tag{3.1.4}$$

Thus $\hat{R}(\bar{x},\ \bar{y})$ is asymptotically unbiased for $R(\lambda,\ \mu)$.

Further, since X and Y are independent hence $\bar{x}$ and $\bar{y}$ are also independent and so

$$Cov(\bar{x},\ \bar{y}) = 0 \tag{3.1.5}$$

Then from Eq.(2.4) and Eq.(3.1.5) we have

$$Var\left[\hat{R}(\bar{x},\ \bar{y})\right] = \left[\frac{\partial \hat{R}(\bar{x},\bar{y})}{\partial \bar{x}}\right]^2_{\bar{x}=\lambda,\bar{y}=\mu} Var(\bar{x}) + \left[\frac{\partial \hat{R}(\bar{x},\bar{y})}{\partial \bar{y}}\right]^2_{\bar{x}=\lambda,\bar{y}=\mu} Var(\bar{y})$$

$$+ O\left(\frac{1}{M^{3/2}}\right) \tag{3.1.6}$$

Now, from Eq.(3.1.3)

$$\left[\frac{\partial \hat{R}(\bar{x},\bar{y})}{\partial \bar{x}}\right] = \frac{\bar{y}}{(\bar{x}+\bar{y})^2} \text{ and } \left[\frac{\partial \hat{R}(\bar{x},\bar{y})}{\partial \bar{y}}\right] = \frac{-\bar{x}}{(\bar{x}+\bar{y})^2} \tag{3.1.7}$$

We also know that for exponential distributions, considered here

Var (X) = λ^2 and Var(Y) = μ^2

Hence, $Var(\bar{X}) = \frac{Var(X)}{M} = \frac{\lambda^2}{M}$ and $Var(\bar{Y}) = \frac{Var(Y)}{M} = \frac{\mu^2}{M}$ (3.1.8)

Substituting from Eq.(3.1.7) and Eq.(3.1.8) in Eq.(3.1.6), after some simplifications, we get

$$Var\left[\hat{R}(\bar{x},\ \bar{y})\right] = \frac{2\lambda^2\mu^2}{M(\lambda+\mu)^4} + O\left(\frac{1}{M^{3/2}}\right), \tag{3.1.9}$$

which →0 as M→ ∞. Hence, $\hat{R}(\bar{x},\ \bar{y})$ is consistent for $R(\lambda,\ \mu)$. Thus $\hat{R}(\bar{x},\ \bar{y})$ is asymptotically unbiased, consistent and sufficient for $R(\lambda,\ \mu)$, since $\bar{x}$, $\bar{y}$ are sufficient for λ and μ, respectively.

From Eq.(3.1.4) and Eq.(3.1.9) we get

$$DM = \left|E\left[\hat{R}(\bar{x},\bar{y})\right] - \frac{\lambda}{\lambda+\mu}\right| \tag{3.1.10}$$

$$DV = \left|Var\left[\hat{R}(\bar{x},\bar{y})\right] - \frac{2\lambda^2\mu^2}{M(\lambda+\mu)}\right| \tag{3.1.11}$$

We use MCS to establish the validity of the approximate expressions Eq.(3.1.4) and Eq.(3.1.9). As earlier, we take independent samples of sizes M from $\exp(\lambda)$ and $\exp(\mu)$, populations for a given λ and μ. From these we obtain estimates of λ and μ as $\bar{x}$ and $\bar{y}$ and substituting these values in Eq.(3.1.4) we get an estimate of $\hat{R}$. The whole process is repeated k times and accordingly we get k values of $\hat{R}$. We obtain mean and variance of $\hat{R}$ viz. $E[\hat{R}(\bar{x},\bar{y})]$ and Var $[\hat{R}(\bar{x},\bar{y})]$. The values are tabulated in Table 3.1.1. We have also drawn normal probability plot (NPP) for $\hat{R}$ in each case and found that the fitting of normal distribution is quite good. It conform finding of (Lloyd and Lipow, 1962). For illustration purpose, we have given a NPP graph for λ= 2, μ= 2, M = 500 and k = 100 in Fig.3.2.1. DM and DV are obtained from Eq.(3.1.10) and Eq.(3.1.11) for given λ, μ and M and given in the same Table 3.1.1.

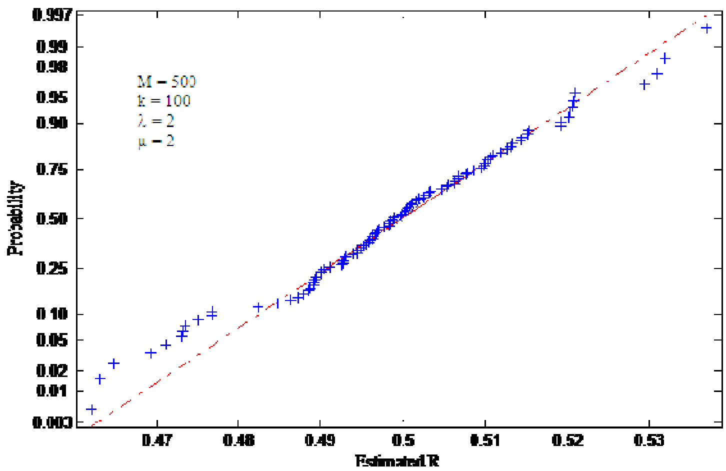

Figure 3.1.1: (Exponential S-S)

Note: In Table 3.1.1, V = $\sqrt{\dfrac{2\lambda^2\mu^2}{M(\lambda+\mu)}}$

Table 3.1.1: Exponential Stress-Strength

M	K	λ	μ	R	Mean of $\hat{R}$	SD of $\hat{R}$	V	Z	DM	DV
500	50	2	2	0.5000	0.5023	0.0139	0.0500	1.3014	0.0006	0.0023
500	200	2	2	0.5000	0.4990	0.0152	0.0250	0.9045	0.0007	0.0004
500	100	2	2	0.5000	0.4992	0.0155	0.0354	0.5232	0.0012	0.0011
500	100	3	2	0.6000	0.6017	0.0156	0.0339	0.0827	0.0011	0.0009
500	100	2	3	0.4000	0.3993	0.0150	0.0339	0.4641	0.0016	0.0009
100	100	2	2	0.5000	0.4962	0.0390	0.0354	0.9792	0.0045	0.0014
100	100	2	3	0.4000	0.3995	0.0328	0.0339	0.1573	0.0041	0.0014
100	100	3	2	0.6000	0.5996	0.0290	0.0339	0.1372	0.0001	0.0012
50	100	2	2	0.5000	0.5029	0.0551	0.0354	1.6772	0.0019	0.0001
50	100	2	3	0.4000	0.4097	0.0478	0.0339	2.0269	0.0006	0.0001
50	100	3	2	0.6000	0.5050	0.0562	0.0339	0.8949	0.0078	0.0004

As noted earlier, for achieving a better estimates of R, we have taken M = 500. But to see the accuracy of Eq.(3.1.4) and Eq.(3.1.9), we have taken M = 50 and 100 also and found that all z-values are insignificant barring M = 50 when k = 100, λ= 2 and μ= 2. The values of DM and DV are quite small for different values of M, so we may assume that approximations given by Eq.(3.1.4) and Eq.(3.1.9) are up to the mark.

3.2 Stress -Strength Are Normal Variates

Let us suppose that X~N(λ, ω^2) and Y~N(μ, σ^2). Then we have seen that the reliability of the system is given by (Kapur and Lamberson, 1977)

$$R = \Phi\left[\frac{(\lambda-\mu)}{\sqrt{(\omega^2+\sigma^2)}}\right], -\infty < \lambda, \mu < \infty;\ \omega^2 > 0,\ \sigma^2 > 0, \tag{3.2.1}$$

where $\Phi(*)$ is c.d.f. of standardized normal variate.

Without loss of generality, we may assume that one of the variable follows N(0, 1). Or often the mean and variance of strength are known because, eg., production is more or less under control. In such case also we may assume that X ~ N(0, 1). Then Eq.(3.2.1) gives,

$$R = \Phi\left[\frac{-\mu}{\sqrt{(1+\sigma^2)}}\right] = 1-\Phi\left[\frac{\mu}{\sqrt{(1+\sigma^2)}}\right] = R(\mu, \sigma^2), \text{ say} \tag{3.2.2}$$

Let us suppose that M components are put on test whose strengths are known and let $y_1, y_2, \ldots, y_M$ be the M stresses on them. We know that $\bar{y}$ and s^2 are independent MLE's of μ and σ^2, respectively, which are unbiased, consistent and sufficient, where

$$s^2 = \sum_{i=1}^{M}\frac{(y_i-\bar{y})^2}{M-1} \text{ and } \bar{y} = \frac{1}{M}\sum_{i=1}^{M} y_i \tag{3.2.3}$$

Then from the properties of MLE's, $\hat{R}$, given below, is the MLE of R

$$\hat{R} = 1-\Phi\left[\frac{\bar{y}}{\sqrt{(1+s^2)}}\right] = \hat{R}(\bar{y}, s^2), \text{ say} \tag{3.2.4}$$

Hence, from Eq.(2.3)

$$E\left[\hat{R}(\bar{y}, s^2)\right] = R(\mu, \sigma^2) + O\left(\frac{1}{M}\right) \tag{3.2.5}$$

Now, since $\bar{y}$ and s^2 are independent, hence

$Cov(\bar{y}, s^2) = 0$. (3.2.6)

Hence from Eq.(2.4) and Eq.(3.2.6) we have

$$\mathrm{Var}\left[\hat{R}(\bar{y}, s^2)\right] = \left[\frac{\partial \hat{R}(\bar{y}, s^2)}{\partial \bar{y}}\right]^2_{\bar{y}=\mu, s^2=\sigma^2} \mathrm{Var}(\bar{y}) + \left[\frac{\partial \hat{R}(\bar{y}, s^2)}{\partial s^2}\right]^2_{\bar{y}=\mu, s^2=\sigma^2} \mathrm{Var}(s^2)$$

$$+\, O\left(\frac{1}{M^{3/2}}\right) \tag{3.2.7}$$

Now from Eq.(3.2.2)

$$\left[\frac{\partial \hat{R}(\bar{y}, s^2)}{\partial \bar{y}}\right] = \left[-\frac{1}{\sqrt{(1+s^2)}}\right] \Phi\left[\frac{\bar{y}}{\sqrt{(1+s^2)}}\right], \tag{3.2.8}$$

$$\text{And} \left[\frac{\partial \hat{R}(\bar{y}, s^2)}{\partial s^2}\right] = \frac{1}{2}\left[\frac{\bar{y}}{(1+s^2)^{3/2}}\right] \Phi\left[\frac{\bar{y}}{\sqrt{(1+s^2)}}\right] \tag{3.2.9}$$

Further, we know that

$$\mathrm{Var}(\bar{y}) = \frac{\sigma^2}{M} \text{ and } \mathrm{Var}(s^2) = \frac{2\sigma^4}{M} \tag{3.2.10}$$

Substituting from Eq.(3.2.8) , Eq.(3.2.9) and Eq.(3.2.7) in Eq.(3.2.10) we get

$$\mathrm{Var}\left[\hat{R}(\bar{y}, s^2)\right] = \frac{1}{1+\sigma^2}\frac{\sigma^2}{M}\Phi^2 + \frac{\mu^2}{4(1+\sigma^2)^3}\frac{2\sigma^4}{M}\Phi^2 + O\left(\frac{1}{M^{3/2}}\right), \tag{3.2.11}$$

Where $\Phi = \Phi\left[\frac{-\mu}{\sqrt{(1+\sigma^2)}}\right]$.

$$\text{Or } \mathrm{Var}\left[\hat{R}(\bar{y}, s^2)\right] \simeq \frac{\Phi^2}{M}\left[\frac{\sigma^2}{1+\sigma^2} + \frac{\mu^2\sigma^4}{2(1+\sigma^2)^3}\right] \tag{3.2.12}$$

Thus, we see from Eq.(3.2.5) that $\hat{R}$ is asymptotically unbiased for R and from Eq.(3.2.11) it is consistent also. Since, $\bar{y}$, s^2 are sufficient for μ and σ^2, so it is sufficient also.

Now, from Eq.(3.2.5) and Eq.(3.2.11),

$$\text{DM}= \left|E\left[\hat{R}(\bar{y}, s^2)\right] - R(\mu, \sigma^2)\right| \tag{3.2.13}$$

$$\text{And DV} = \left|\mathrm{Var}[\hat{R}(\bar{y}, s^2)] - \frac{1}{1+\sigma^2}\frac{\sigma^2}{M}\Phi^2 - \frac{\mu^2}{4(1+\sigma^2)^3}\frac{2\sigma^4}{M}\Phi^2\right| \tag{3.2.14}$$

Here, also we use MCS for the validation of the approximations Eq. (3.2.5) and Eq.(3.2.11). For this as in the

Sec.3.1, we take samples of different sizes M from $N(\mu, \sigma^2)$ population for particular values of μ and σ^2; its mean and variance are $\bar{y}$ and s^2, respectively. Substituting these values in Eq.(3.2.4), we get an estimate $\hat{R}$ of R. The whole process is repeated k times giving k values of $\hat{R}$. Its mean and variance gives $E[\hat{R}(\bar{y}, s^2)]$ and $\mathrm{Var}[\hat{R}(\bar{y}, s^2)]$, respectively. DM and DV are obtained from Eq. (3.2.13) and Eq.(3.2.14). The values are tabulated in Table 3.2.1. We have also drawn NPP graphs for $\hat{R}$ in each case of Table 3.2.1. NPP graphs suggest that the data sets reasonably follow the normal distribution. For illustration purpose we have given only one NPP graph of $\hat{R}$ when M = 500, k = 200, μ = 1 and σ = 1 in Figure 3.2.1.

Here, also we have taken M = 500, but for checking the accuracy of Eq.(3.2.5) and Eq.(3.2.11), we have taken M = 50 and 100 also. From Table 3.2.1, we note that for k = 100, z-value is significant. So for achieving a better estimate of R we have taken k = 200 throughout and found that all z-values are insignificant. For example, z = 0.2522 when m = 0 and σ = 1. We observe that the values of DM and DV are quite small for different values of M, so the approximations given by Eq. (3.2.5) and Eq. (3.2.11) are reasonable.

Table 3.2.1: Normal Stress-Strength

M	K	μ	σ	R	Mean of $\hat{R}$	SD of $\hat{R}$	Z for $\hat{R}$	DM	DV
500	100	0	1	0.5000	0.4975	0.0115	2.2126	0.0002	0.0001
500	200	0	1	0.5002	0.5002	0.0122	0.2522	0.0002	0.0001
500	200	1	1	0.2398	0.2400	0.0116	0.2822	0.0002	0.0001
500	200	2	1	0.0787	0.0787	0.0056	0.1737	0.0000	0.0000
500	200	-1	1	0.7603	0.7608	0.0113	0.7002	0.0007	0.0006
500	200	-2	1	0.9214	0.9211	0.0057	0.6402	0.0001	0.0011
100	200	0	1	0.5000	0.4986	0.0277	0.7397	0.0016	0.0010
100	200	1	1	0.2398	0.2403	0.0252	0.3223	0.0042	0.0006
100	200	2	1	0.0787	0.0781	0.0125	0.6477	0.0029	0.0027
100	200	-1	1	0.7603	0.7574	0.0220	1.8355	0.0002	0.0056
100	200	-2	1	0.9214	0.9202	0.0128	1.2910	0.0020	0.0124
50	200	0	1	0.5000	0.5000	0.0376	0.3851	0.0027	0.0005
50	200	1	1	0.2398	0.2402	0.0343	0.1731	0.0013	0.0002
50	200	2	1	0.0787	0.0767	0.0176	1.5687	0.0004	0.0001
50	200	-1	1	0.7603	0.7590	0.0323	0.5649	0.0005	0.0027
50	200	-2	1	0.9214	0.9203	0.0189	0.7611	0.0011	0.0062

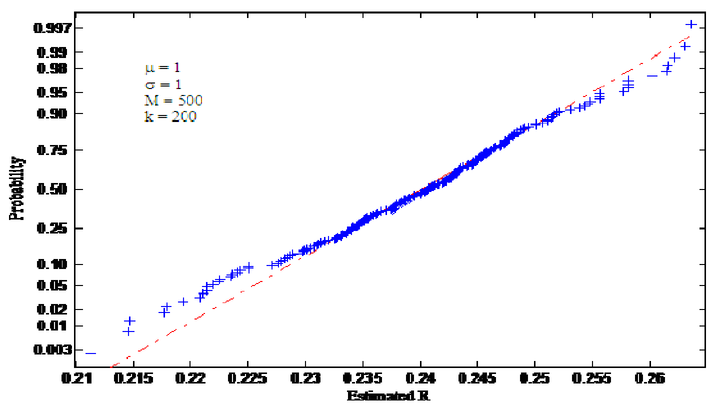

Figure 3.2.1: (Normal S-S)

4. CONCLUSIONS

In this paper, approximate expressions for mean and variance of estimated reliability in interference models are evaluated using (Lloyd and Lipow, 1962) method. Here, we have considered two cases-when both stress and strength follow exponential distribution and when both stress and strength follow normal distribution. Validity of approximations is checked by Monte-Carlo simulation. We have found that expressions are acceptable, particularly when the sample size is reasonably large.. The approximate expressions can be extended when stress strength follow more than two parameters distributions, also.

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